mathemagix

Class XII - Math

Chapter: Differential Calculus

Concepts and Formulae

S.No	Chapter		Formula
1	**Continuity & Differentiability**	**1.1**	**Continuity of a function** ▪ A function f(x) is said to be continuous at a point c if, $\lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$
		1.2	**Algebra of Continuous Functions** If f and g are continuous functions, then ▪ $(f \pm g)(x) = f(x) \pm g(x)$ is continuous ▪ $(f.g)(x) = f(x).g(x)$ is continuous ▪ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ (where $g(x) \neq 0$) is continuous
		1.3	**Differentiability of a function** ▪ A function f is differentiable at a point c If, LHD=RHD i.e $\lim_{h\to 0^-} \frac{f(c+h)-f(c)}{h} = \lim_{h\to 0^+} \frac{f(c+h)-f(c)}{h}$ ▪ Derivative of a function f is f′(x) which is $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ ▪ Every differentiable function is continuous, but converse is not true.
		1.3	**Algebra of Derivatives** If u & v are two functions which are differentiable, then ▪ $(u \pm v)' = u' \pm v'$ ▪ $(uv)' = u'v + uv'$ (Product rule) ▪ $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$ (Quotient rule)
		1.4	**Derivatives of Functions** ▪ $\frac{d}{dx}x^n = nx^{n-1}$

			▪ $\frac{d}{dx}(\sin x) = \cos x$ ▪ $\frac{d}{dx}(\cos x) = -\sin x$ ▪ $\frac{d}{dx}(\tan x) = \sec^2 x$ ▪ $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$ ▪ $\frac{d}{dx}(\sec x) = \sec x \tan x$ ▪ $\frac{d}{dx}(\mathrm{cosec} x) = -\mathrm{cosec} x \cot x$ ▪ $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$ ▪ $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$ ▪ $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$ ▪ $\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$ ▪ $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$ ▪ $\frac{d}{dx}(\mathrm{cosec}^{-1} x) = \frac{-1}{x\sqrt{x^2-1}}$ ▪ $\frac{d}{dx}(e^x) = e^x$ ▪ $\frac{d}{dx}(\log x) = \frac{1}{x}$
		1.5	**Chain Rule** If f = v o u, t = u(x) & if both $\frac{dt}{dx}$ and $\frac{dv}{dx}$, exists then, $\frac{df}{dx} = \frac{dv}{dt} \cdot \frac{dt}{dx}$
		1.6	**Implicit Functions** If it is not possible to "separate" the variables x & y then function f is known as implicit function.

1.7	**Logarithms** $\log(xy) = \log x + \log y$ $\log\left(\frac{x}{y}\right) = \log x - \log y$ $\log\left(x^y\right) = y \log x$ $\log_a x = \frac{\log_b x}{\log_b a}$
1.8	**Logarithmic Differentiation** Differentiation of $y=a^x$ Taking logarithm on both sides $\log y = \log a^x$. Using property of logarithms $\log y = x \log a$ Now differentiating the implicit function $\frac{1}{y}.\frac{dy}{dx} = \log a$ $\frac{dy}{dx} = y \log a = a^x \log a$
1.9	**Parametric Differentiation** Functions of the form x = f(t) and y = g(t) are parametric functions. $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$
1.10	**Mean Value Theorems** ▪ **Rolle's Theorem**: If f : [a, b] → **R** is continuous on [a, b] and differentiable on (a, b) such that f (a) = f (b), then there exists some c in (a, b) such that f′(c) = 0 ▪ **Mean Value Theorem**: If f :[a, b] → **R** is continuous on [a, b] & differentiable on (a, b). Then there exists some c in (a, b) such that $f'(c) = \lim_{h\to 0} \frac{f(b) - f(a)}{b - a}$

2	**Application of derivatives**	**2.1**	**Increasing & Decreasing functions** Let I be an open interval contained in domain of a real valued function f. Then f is said to be: ▪ Increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \le f(x_2)$ for all $x_1, x_2 \in I$ ▪ Strictly increasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) < f(x_2)$ for all $x_1, x_2 \in I$ ▪ Decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) \ge f(x_2)$ for all $x_1, x_2 \in I$ ▪ Strictly decreasing on I if $x_1 < x_2$ in I $\Rightarrow f(x_1) > f(x_2)$ for all $x_1, x_2 \in I$ **Theorem:** Let f be a continuous function on [a,b] and differentiable on (a,b).Then (a)f is increasing in[a,b] if $f'(x)>0$for each $x\in(a,b)$ (b) f is decreasing in[a,b] if $f'(x)<0$ for each $x\in(a,b)$ (c) f is constant in[a,b] if $f'(x)=0$ for each $x\in(a,b)$
		2.3	**Tangents & Normals** ▪ The equation of the tangent at (x_0, y_0) to the curve $y = f(x)$ is: $y - y_0 = f'(x_0)(x - x_0)$ ▪ Slope of a tangent = $\frac{dy}{dx} = \tan\theta$ ▪ The equation of the normal to the curve $y = f(x)$ at (x_0, y_0) is: $(y-y_0)f'(x_0)+(x-x_0)= 0$ ▪ Slope of Normal $= \frac{-1}{\text{slope of the tangent}}$
		2.4	**First Derivative Test** Let f be a function defined on an open interval I. Let f be continuous at a critical point c in I. Then ▪ If $f'(x) > 0$ at every point sufficiently

			close to and to the left of c & $f'(x) < 0$ at every point sufficiently close to and to the right of c, then c is a point of local maxima. ▪ If $f'(x) < 0$ at every point sufficiently close to and to the left of c, $f'(x) > 0$ at every point sufficiently close to and to the right of c, then c is a point of local minima. ▪ If $f'(x)$ does not change sign as x increases through c, then point c is called point of inflexion
		2.5	**Second Derivative test** Let f be a function defined on an interval I & $c \in I$. Let f be twice differentiable at c. Then ▪ $x = c$ is a point of local maxima if $f'(c) = 0$ & $f''(c) < 0$. ▪ $x = c$ is a point of local minima if $f'(c) = 0$ and $f''(c) > 0$ ▪ The test fails if $f'(c) = 0$ & $f''(c) = 0$. By first derivative test, find whether c is a point of maxima, minima or a point of inflexion.
		2.6	**Differential Approximations** ▪ Let $y = f(x)$, Δx be small increments in x and Δy be small increments in y corresponding to the increment in x, i.e., $\Delta y = f(x+\Delta x) - f(x)$. Then $\Delta y = \left(\frac{dy}{dx}\right)\Delta x$ or $dy = \left(\frac{dy}{dx}\right)\Delta x$ $\Delta y \approx dy$ and $\Delta x \approx dx$